Adaptive Sampling of Physical Optics Currents Based on EFIE Error Prediction

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Simple and efficient method for a numerical analysis of an electromagnetic scattering problem is developed by converting the electric field integral equation (EFIE) into a simple integral equation with known variables. Regarding electrically large conducting objects, the surface current density can easily be approximated by the physical equivalent principle. The combination of the equivalent surface current and the EFIE generates unexpected tangential field component right on the surface, which is against the boundary condition. This numerical error can be used to define abnormality of the object, and the integration for the field radiation must be performed carefully, especially over the abnormal surfaces. In this paper, an adaptive sampling of PO currents is proposed. The resulting radar cross section (RCS) is compared to the results from the Method of Moments (MoM) as well as the general PO.

***Index Terms*—Estimation error, Electromagnetic fields, Integral equations, Physical optics**

I. Introduction

PHYSICAL OPTICS (PO) is one of the most well-known techniques for analyzing high frequency electromagnetic fields scattered by electrically large conducting objects. Due to the fact that a flat perfect electric conductor (PEC) of infinite extent is assumed for the PO formulation, the evaluation of the PO current is limited either on the unsmooth surface or near the edge discontinuity [1].

Increasing the resolution of sampling points up to some fraction of the wavelength can be one of the simplest ways to reduce an overall error, but computational cost rises dramatically for the electrically large scatterer. However, while minimizing the increase in the number of sample points, the accuracy of scattered fields can be improved by arranging them to appropriate locations so that the sampling resolution can differ by an order of abnormality of the surface.

In this paper, a method for an adaptive sampling of the PO surface current is presented to analyze complex scattering problem. The arrangement of the evaluation points is performed based on an error derived from the electric field integral equation (EFIE). The resulting radar cross section (RCS) is compared to the results from the other numerical technique as well as the general PO.

II. Formulations

A. Physical Optics

The current density generated from a known incident field on a surface of PEC can be approximated by PO. In the frequency domain, the PO current is

$$\mathbf{J}_{PO} = 2\hat{\mathbf{n}} \times \mathbf{H}^i \tag{1}$$

where $\hat{\mathbf{n}}$ is the unit normal vector pointing out of the surface, and $\mathbf{H}^i$ is the incident magnetic field. Since the PO current is a function of an incident field, a zero surface current is assumed in so-called shadow regions. The scattered fields from the surface currents can be obtained as follows,

$$\mathbf{A} = \frac{\mu}{4\pi}\iint_S \mathbf{J}_{PO}\frac{e^{-jkr}}{r}d\mathbf{r}' \tag{2}$$

$$\mathbf{E}^s = -j\omega\mathbf{A} - j\frac{1}{\omega\mu\varepsilon}\nabla(\nabla\cdot\mathbf{A}) \tag{3}$$

where $\mathbf{A}$ is a magnetic vector potential, $\mathbf{E}^s$ is the scattered field, and r is the distance between the observation (unprimed) and source (primed) coordinates.

B. Electric Field Integral Equation

The three-dimensional EFIE for a conducting surface of arbitrary shape is

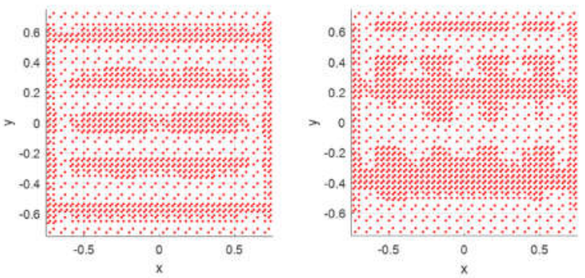
$$-\frac{j}{\omega\mu}\hat{\mathbf{t}}(\mathbf{r})\cdot\mathbf{E}^i(\mathbf{r}) = \hat{\mathbf{t}}(\mathbf{r})\cdot\iint_S(1+\frac{1}{k^2}\nabla\nabla\cdot)\mathbf{J}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')d\mathbf{r}' \tag{4}$$

where $G(\mathbf{r},\mathbf{r}') = \exp(-jkr)/(4\pi r)$ is the three-dimensional Green's function with $r = |\mathbf{r}-\mathbf{r}'|$, $\mathbf{E}^i(\mathbf{r})$ is the excitation, $\mathbf{J}(\mathbf{r}')$ is the unknown current density, and $\hat{\mathbf{t}}(\mathbf{r})$ is the unit tangential vector. The EFIE is based on the boundary condition that the total tangential electric field on a surface of PEC is zero. In general, the unknown $\mathbf{J}(\mathbf{r}')$ is to be determined by solving EFIE with the known excitation.

C. Error Prediction

Instead of solving the EFIE for the unknown $\mathbf{J}(\mathbf{r}')$, which requires the matrix inversion process, the PO current density is to be inserted into (4). The modified EFIE can be written as

$$\hat{\mathbf{t}}(\mathbf{r})\cdot\iint_S(1+\frac{1}{k^2}\nabla\nabla\cdot)\mathbf{J}_{PO}(\mathbf{r}')G(\mathbf{r},\mathbf{r}')d\mathbf{r}' + \frac{j}{\omega\mu}\hat{\mathbf{t}}(\mathbf{r})\cdot\mathbf{E}^i(\mathbf{r}) \neq 0 \tag{5}$$



(a) (b)

Fig. 1. Point distribution with incident plane wave of: (a) $\theta_i = 0°$. (b) $\theta_i = 30°$.

The left hand side of (5) can then be solved by simply integrating the known quantities over the surface. In this case, the scattered fields are evaluated right on the surface $(r = r_s)$. Since the PO current density is approximated by (1), the total tangential electric field on the surface cannot be zero, which allows us to define an error,

$$\varepsilon_{abs} = \left|\mathbf{E}_t^s + \mathbf{E}_t^i\right| \quad (6)$$

where $\mathbf{E}_t^s$ and $\mathbf{E}_t^i$ are represent the first term and the second term on the left hand side of (5), respectively.

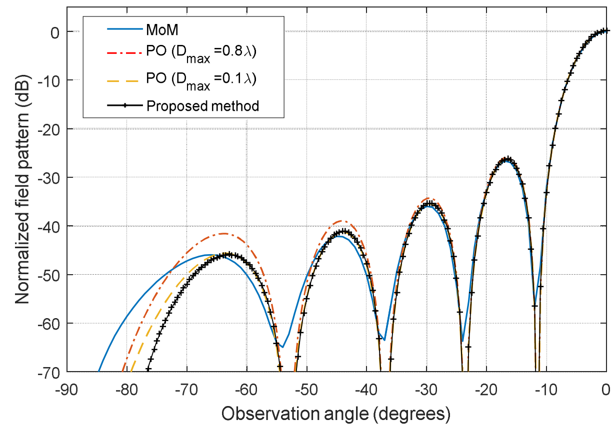
The boundary condition cannot be satisfied due to the incorrect PO currents, and the amount of error at any observation point on the surface is simply the magnitude of the total tangential electric field. Unfortunately, we are not interested in this numerical error. Instead, this absolute error ε_{abs} can be utilized in adaptive sampling of the PO currents. For example, by comparing the magnitude of ε_{abs} on each triangular element, one can determine the order of subdivision up to four smaller triangles.

D. Integration Technique

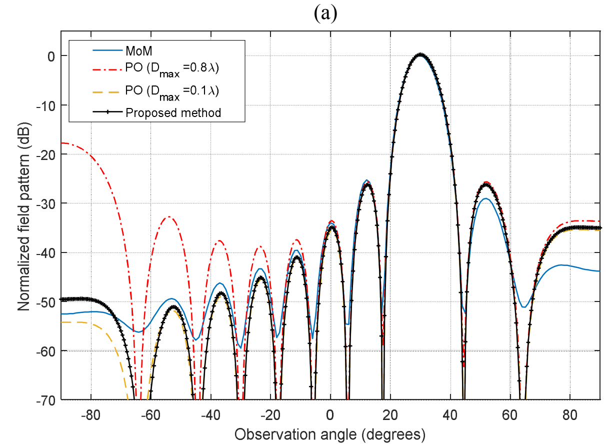
The three-dimensional surface of a scattering object of arbitrary shape is discretized into planar triangles. The integrations in the previous section will be performed on these triangles based on the Rao-Wilton-Glisson (RWG) triangular basis function [2]. Since the observation points are laid on the surface where the PO currents exist, the integrands become singular and need to be integrated analytically. In this case, the singularity extraction technique is used to deal with the Green's function [3]-[4].

III. Numerical Results

To verify the validity of the proposed approach, we assume the plane wave with the TM^x polarization that normally or obliquely incident on a conducting square plate on the *xy* plane with a side of 5λ at $f = 1GHz$. The sampling points for the PO currents in Fig. 1 are adjusted according to the absolute error of (6). The magnitude of each PO current remains constant regardless of the changed location. However, the adjusted sampling points build up the entire region with newly generated triangles that allows the surface integration for the scattered field to be more focused on the region with a larger absolute error.



(a)



(b)

Fig. 2. Normalized field pattern for different methods:
(a) $\theta_i = 0°$. (b) $\theta_i = 30°$.

In Fig. 2, the normalized field patterns from the proposed method are presented and compared to the result from the MoM and PO with maximum distance between sampling points D_{max} . Since the proposed method is based on pure PO and operates without any help of IPO or PTD, the pattern is distorted when the field is observed at an angle close to $\pm 90°$. However, the field pattern can be significantly improved by the proposed method without sampling every 0.1λ . For further verification, the proposed method will be applied to the other objects such as a circular disk, a sphere, and a bent plate.

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